TECHNICAL NOTE

An Improved Tree Height Measurement Technique Tested on Mature Southern Pines

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ABSTRACT

Virtually all techniques for tree height determination follow one of two principles: similar triangles or the tangent method. Most people apply the latter approach, which uses the tangents of the angles to the top and bottom and a true horizontal distance to the subject tree. However, few adjust this method for ground slope, tree lean, crown shape, and crown configuration, making errors commonplace. Given documented discrepancies exceeding 30% with current methods, a reevaluation of height measurement is in order. The sine method is an alternative that measures a real point in the crown. Hence, it is not subject to the same assumptions as the similar triangle and tangent approaches. In addition, the sine method is insensitive to distance from tree or observer position and can not overestimate tree height. The advantages of the sine approach are shown with mature southern pines from Arkansas.

Keywords: hypsometer, sine method, tangent method

Foresters have measured height in many ways since the earliest years of the profession (e.g., Schlich 1911). One technique, direct measurement, is commonly done using a height pole for small- to medium-sized trees, and rarely with a tape dropped from the top of tall trees. Direct measurement, however, typically requires the observer to carry a bulky height pole or be skilled in the hazardous art of tree climbing. Most other height measurement techniques use mathematics as their basis. For example, some have used a pole or rod of known length as the basis for comparison with a standing tree (e.g., Curtis and Bruce 1968, Bell and Gourley 1980). This proportionality approach reduces the size of the pole needed to determine height but provides only an approximation of height. Hypsometers also have been developed with more sophisticated optically based mathematics or digital image processing using other proportional formulations for height determination (Anuchin 1971, Clark et al. 2000).

None of these approaches, however, have supplanted the most popular height measuring techniques. One of the oldest applies the geometry of similar triangles, which operates on the principle that triangles of the exact same configuration are direct scalars of each other. The most common height determination approach can be called the "tangent method" and multiplies the tangent of the angles to the top and bottom of the crown with a true horizontal baseline distance from the observer to the tree. Under idealized circumstances, both similar triangles and the tangent method will yield the exact height of a tree. However, rarely are the assumptions of either satisfied, and the resultant measurement errors can be significant. For instance, the Eastern Native Tree Society (ENTS) has documented numerous cases of national champion-sized trees measured with these techniques being overestimated by 5–20 m (ENTS 2006).

Fortunately, the increasing availability of inexpensive and accurate distance measuring devices has provided a unique solution for most measurement problems. A "new" trigonometric approach to height determination uses the product of the sine of the angles to the top and bottom of a tree and their respective slope distances to determine height. The "sine method" was developed originally to improve height estimates of very large individual trees and has been repeatedly validated using direct measurements with tapes lowered from very tall (>50 m) specimens (Blozan 2006). In this article, advantages of the sine method with three mature pines from southern Arkansas are shown.

Methods and Materials

The Tangent and Sine Height Determination Methods

Any height computation technique assumes that all angles and distances are measured without error and that the observer can identify the highest part of the crown. The best means to differentiate between the sine and tangent methods of height determination is to display the techniques graphically. The tangent method must either meet some very precise assumptions or be correctly adjusted to produce the exact height of a tree. Specifically, these assumptions are the tree must be truly vertical (i.e., perpendicular to a level horizontal plane), the diameter of a tree can be ignored, and the highest point of the live crown is located directly over the base of the stem. The tangent method shown in Figure 1a can be expressed as

$$HT = b \times \tan(A) + b' \times \tan(A'), \quad (1)$$

where the baseline lengths (b and b', identical in this case) are true horizontal distances and the angles A and A' are of the true top and bottom of the tree, respectively. Note that under these same circum-

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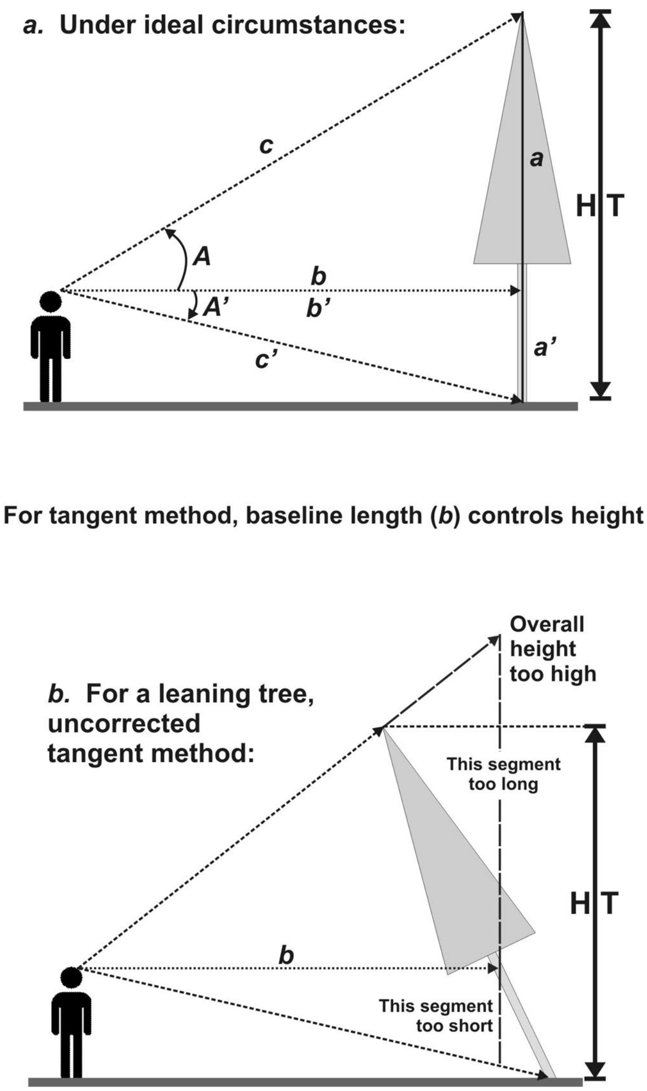

Figure 1. The tangent method of tree height determination, under (a) ideal and (b) less-than-ideal circumstances.

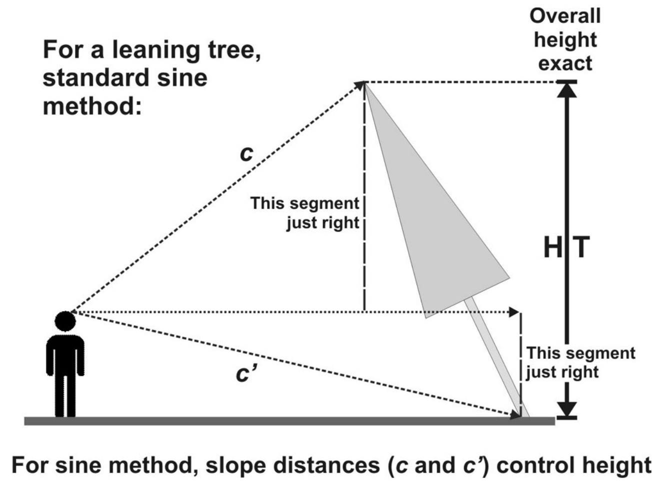

Figure 2. The sine method addresses leaning trees or offset crowns by directly measuring the slope distance to the top (c) and bottom (c') of the tree.

stances, the following is also true:

$$HT = c \times \sin(A) + c' \times \sin(A'), \qquad (2)$$

where c and c' are the slope distances corresponding to A and A', respectively. Equation 2 is the standard form of the sine method and would exactly equal the tangent-determined height under the conditions shown in Figure 1a.

However, for the tangent method to yield the actual height of a leaning tree or one with an offset crown, corrections are needed to ensure that the proper baseline distances are used (Figure 1b). Under less-than-ideal conditions, the tangent method can overestimate height if the lean or offset is toward the observer or underestimate height if the lean/offset trends away (see also Husch et al. 2003). Typically, adjustments to baseline length or perspective are made to reduce this error—either the observer measures horizontal distance to a spot assumed to be directly under the highest point or the observer moves to a position perpendicular to the lean or crown offset tree to ensure that any departure from the true base is minimized. Although both corrections can help, rarely are they precise enough to eliminate estimation problems, and either can prove time-consuming or even impossible.

The sine method is free of virtually every assumption of the other techniques because it directly measures a point on the tree crown, not a projection based on a potentially erroneous baseline distance. Measuring the requisite slope length (Figure 2) has been made possible only recently by the advent of accurate technology (e.g., laser rangefinders). To be effective, the slope measuring device must be able to strike the correct part of the tree, which can be challenging on windy days or in thick vegetation. Because it does not need to be performed from any specific location providing the true top and bottom of the tree are visible, the sine method permits the observer to move around the tree until the best vantage point is found. Hence, there is no need to be perpendicular to a leaning or offset crown top, as required by the most common tangent method correction applied in the field, or to spend time identifying the crown nadir.

Test Subjects and Study Area

Three mature pines were selected for this study. One was an 89-cm dbh loblolly pine (*Pinus taeda* L.) from the Crossett Experimental Forest (CEF) in Ashley County, Arkansas. Second was a 91-cm dbh shortleaf pine (*Pinus echinata* Mill.) from the Levi Wilcoxon Demonstration Forest (LWDF), also in Ashley County. Both of these were on level sites and possessed a slight (4–5°) lean that has been suggested as inconsequential enough to be ignored when determining height (e.g., Avery and Burkhart 1994). The third tree, a 49-cm dbh shortleaf pine located on the University of Arkansas–Monticello (UAM) campus in Drew County, was chosen to highlight the relative insensitivity of the sine method to violations of the assumptions of the tangent method. This shortleaf pine grows on a moderate (approximately 10%) slope, has a spiraling 5–10% lean, and is found in a relatively closed canopy, limiting the number of clear observation points. The close proximity of its crown to its neighbors, coupled with its tilt and the slope of the ground, therefore makes determining the best measuring location difficult.

Measurement Tools

An Impulse 200LR laser rangefinder was used to measure the height of all trees. According to the manufacturer, the Impulse 200LR has a maximum distance measurement error of 15 cm up to

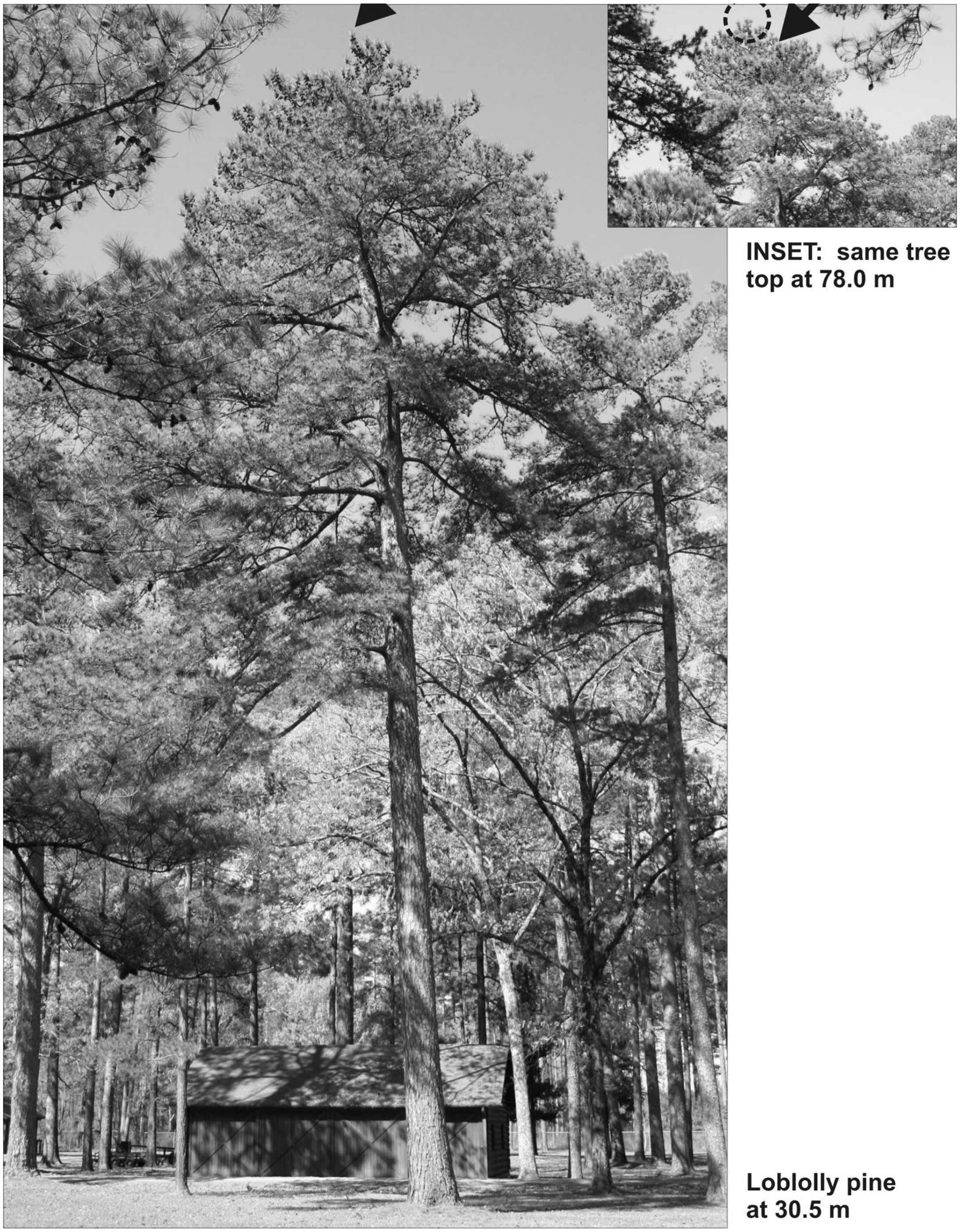

Figure 3. The mistaken identification (arrows) of a subordinate branch as the top of the loblolly pine on the CEF at a distance of 30.5 m was only recognized with the sine method. At 78.0 m, the true top (circled in inset) was visible and clearly taller than the "obvious" top.

575 m (with "typical" accuracy of 3–5 cm), and an angle measurement accuracy of ±0.1° (Laser Technology, Inc. 2006). Because it has an electronic inclinometer and can produce both horizontal and slope distances, this device is capable of both the tangent (the default program) and the sine height techniques.

However, most foresters use more affordable devices such as handheld clinometers and cloth or steel measuring tapes. Even though less accurate, these tools are more compact and can quickly yield a height. For the CEF and LWDF pines, the Impulse provided the exact horizontal distance for the tangent method using both the clinometer and the laser rangefinder. The UAM shortleaf pine was approached somewhat differently—to highlight the potential for multiple errors on the estimation of height, distance from the observer was determined exclusively with a cloth tape for all tangent-based measurements.

Study Design and Analysis

For the Ashley County pines, three to four transects with three observation points each were extended from each tree. Transects corresponded to cardinal directions based on tree lean, so each pine had one transect that followed, one that went against, and at least one perpendicular to the lean. The first observation point along each

Table 1. Height measurements on the CEF loblolly pine taken from three different perspectives (a building blocked the view from the west) using a laser rangefinder (tangent and sine methods) and a tape and clinometer (tangent method only).

	North distance			East distance			South distance					
Measure	30.5	47.0	67.9	30.5	48.2	78.1	30.5	51.6	85.3	Min.	Max.	SD
	(m)											
Tangent	29.4	29.8	29.7	31.2	30.0	30.4	32.9	31.8	31.2	29.4	32.9	1.16
Sine	29.6	29.8	29.9	27.6	30.3	**30.7**	30.0	30.0	30.3	27.6	30.7	0.87
Observer 1	29.3	29.6	29.9	30.2	29.9	30.4	32.0	32.0	32.4	29.3	32.4	1.18
Observer 2	29.3	29.2	28.5	29.9	29.4	29.7	32.6	33.5	31.6	28.5	33.5	1.74
Observer 3	29.6	29.2	29.2	29.9	29.4	29.7	32.6	30.9	32.4	29.2	32.6	1.35

Bold font indicates the greatest sine height.

Table 2. Height measurements for the LWDF shortleaf pine taken from four different perspectives using a laser rangefinder (tangent and sine methods) and a tape and clinometer (tangent method only).

	North distance			East distance			South distance			West distance					
	30.5	47.3	58.8	30.5	46.8	65.3	30.5	53.2	67.0	30.5	45.6	51.7	Min.	Max.	SD
	(m)														
Tangent	42.2	41.9	41.3	40.2	40.5	41.0	43.5	42.1	41.9	46.9	44.0	43.5	40.2	46.9	1.86
Sine	41.1	**41.4**	41.1	39.4	41.2	41.3	38.2	40.3	41.3	39.7	40.5	40.5	38.2	41.4	0.98
Observer 1	41.5	42.1	41.7	39.6	40.7	41.1	43.0	40.5	41.5	46.6	43.8	41.9	39.6	46.6	1.82
Observer 2	40.8	40.7	41.7	40.8	39.8	41.1	43.3	42.1	40.8	46.3	44.3	43.5	39.8	46.3	1.88
Observer 3	41.8	41.6	42.3	39.9	40.3	40.5	42.7	41.5	40.8	46.3	43.8	43.5	39.9	46.3	1.81

Bold font indicates the greatest sine height.

transect was placed at a convenient standard of 30.5 m, and the others were located at good viewpoints from 45 to 85 m along the transects.

Because the UAM pine was included to show the insensitivity of the sine method to poorly chosen locations, a different design was used. Six observation points were chosen where the top and bottom of the pine were clearly visible, regardless of position relative to tree lean, distance from the stem, or ground slope. Because this shortleaf, although noticeably leaning, was not drastically off center, these point locations should emulate a measurer opting for viewing convenience rather than the spot capable of the greatest height accuracy. Each of these locations where chosen because they provided reasonable views of what appeared from each spot to be the top. These points also were relatively close to the subject tree (between 20 and 60 m).

Two to three observers with 5–30 years of forestry experience used a percent-baseline distance clinometer to estimate tree height following conventional techniques at every observation point. Observers were instructed to regard the top of the crown as the highest apparent point from the viewing location. At each of the 6–12 observation points per tree, observers also made tangent and sine height estimates using the laser rangefinder to the exact same point of the crown. To avoid biasing the results of the clinometer measurements, each observer silently and independently recorded their estimated heights.

True height was then needed to compare measured heights. Because we could not drop a tape vertically to directly measure height, the next best option was chosen. The sine method, when applied to the appropriate top and bottom of a tree, can not overestimate tree height. It is possible for the sine height of a subordinate part of the crown to be less than the actual height, especially when the true top of the tree is hidden. Hence, the maximum sine height taken from multiple distances and angles provides a robust estimate of total height. However, the same can not be said for the tangent method, because there is no way to account for how much (or even why) the heights depart from the true value.

Results and Discussion

Both techniques are considerably easier and quicker on open-grown trees. In a forest, the tangent method generally is faster, because one only needs to approximate the top of the crown. However, this convenience lends itself to potentially significant errors in height determination.

Height Accuracy and Variation between the Techniques

If the corresponding angles and distances are measured correctly, the sine method is the most consistent and accurate means to remotely measure tree height because it always measures a physical object rather than a projection of the top of the crown. For the loblolly pine on the CEF, the maximum sine method height estimate is 30.7 m, the shortleaf pine at the LWDF reached 41.4 m, and the shortleaf pine on the UAM campus was 32.0 m tall—these numbers were used as the truest available estimates of their respective total tree heights. To accurately determine total tree height with the tangent method, corrections for leaning stems or offset crown tops must be used. Otherwise, the horizontal baseline difference (Figure 1b) does not reflect either the top or the bottom of the tree as projected. Hilly topography can further exacerbate this problem, especially when using equipment that does not automatically compensate for slope.

Slope was not an issue for the loblolly pine on the CEF. For this south-leaning example, the east-based measurements corresponded best to the true height of the tree. This set of measurements showed very little difference as a group, with one noticeable exception. From the east at a distance of 30.5 m, the tangent method produced height estimates ranging from 29.9 to 31.2 m. This range turns out to be a reasonable approximation of the tree's best height estimate (30.7 m), but for the wrong reason—at this distance, a stray branch projecting toward the observers appeared as the highest point. In reality, this was subordinate to the actual crown top (Figure 3). Only the sine method showed that the elevation of this point (27.6 m) was substantially less than the true height of the loblolly. Note that the quasi-agreement of some of the errant tangent readings in Table 1

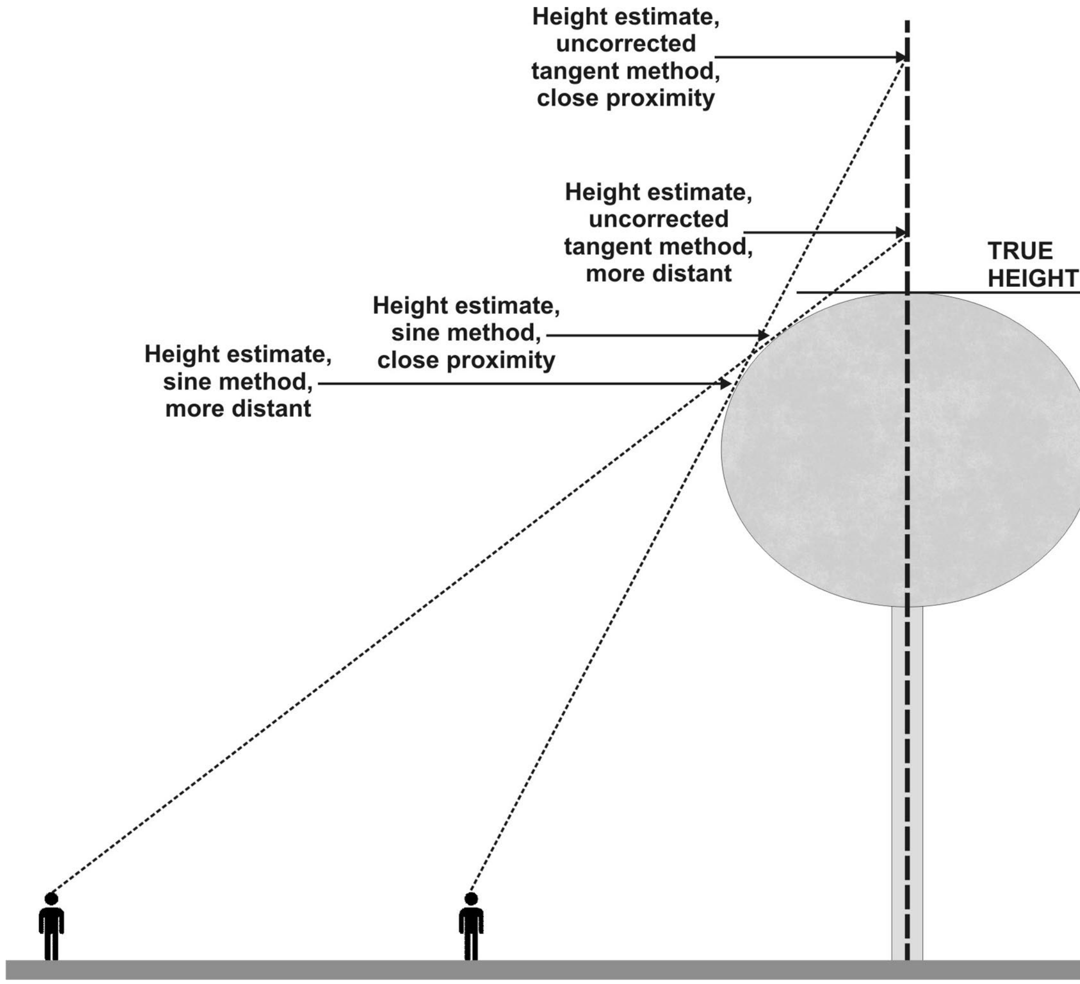

Figure 4. Proximity effects on misread crown heights for the sine and tangent methods.

and true tree height are no guarantee that this pattern will be consistent, given their sensitivity to the assumptions inherent in the technique.

Not surprisingly, the sine method, being insensitive to the primary sources of measurement error using the tangent method (especially tree lean and offset crowns), had noticeably less variation in the estimated heights of the CEF and LWDF pines (Tables 1 and 2). Both trees had a particular direction for which the tangent error was greatest, with departures of 2.5–5.5 m from true height. This may not seem like much, except that neither tree had a dramatic lean or highly offset crown.

The sine method yields a significantly lower degree of variation (standard deviation [SD] = 0.87 m) than any of the tangent-based observations (SD between 1.16 and 1.74 m) of the loblolly pine on the CEF (Table 1). If we ignore the observation of the subordinate branch at 30.5 m, variation is even lower for the sine method (only 0.33 m), and the tangent-based observation SD actually increased slightly (now 1.23–1.84 m). The trends are similar for the shortleaf pine on the LWDF (Table 1), with the sine method yielding almost one-half the SD of the tangent-based observations (0.98 m versus 1.81–1.88 m). Assuming the inherent error of the equipment is negligible, the source of this variation is easily explained for the sine-based observations—the target struck by the laser as the presumed top of the tree was actually a somewhat subordinate position. For the tangent observations, the variation could have resulted from a number of factors, including the lean of the tree or offsets in the top of the crown.

The magnitude of the tangent-based errors also depends on the proximity to the stem—the more distant the observer, the lower the impact of errors in crown position and angle projections. In mature forests of tall stature, proximity becomes increasingly important for tangent-based measures of tree height. As stand density and structural complexity increases, there are more obstructions affecting the view of any given tree top. It becomes virtually impossible to distinguish one from another, especially in monospecific stands. Under these circumstances, it is common for the observer to guess where the tops and/or bottoms of the subject tree are, which can lead to serious misapplications and substantial measurement errors.

Alternatively, observers sometimes adjust for poor visibility by measuring height from very close to the tree, which can result in an additional problem—systematic overestimation (e.g., Husch et al. 2003, Bragg 2007). For instance, many species (especially open-grown hardwoods) produce spreading, dense crowns. If an observer takes the side of the crown as the true top, then errors in height may arise because of an inappropriate baseline distance (Figure 4). The closer the observer is to the subject tree, the greater the error when using the tangent method. The sine method is relatively insensitive to distance from the tree because it measures slope distance of an actual point and accurately yields its height—which, obviously, is still in error, because it is not the intended total tree height (Figure 4).

Table 3. Height measurements of the University of Arkansas–Monticello shortleaf pine taken from six different points surrounding the tree using a laser rangefinder (tangent and sine methods) and a tape and clinometer (tangent method only).

	Point number									
Measure	1	2	3	4	5	6	Min.	Max.	Ave.	SD
Azimuth (°)	53	179	262	290	354	5	....	(m)	....	
Horizontal distance (m)	20.9	34.4	21.9	56.0	22.0	42.1				
Tangent	35.8	31.0	33.3	32.7	37.8	34.9	31.0	37.8	34.3	2.42
Sine	**32.0**	**32.0**	27.8	30.2	31.8	31.9	27.8	32.0	31.0	1.69
Observer 1	33.5	31.4	32.4	34.2	34.6	34.7	31.4	34.7	33.5	1.32
Observer 2	33.7	30.8	32.8	32.0	36.0	34.7	30.8	36.0	33.3	1.88

Bold font indicates the greatest sine height.

The tangent method can be as reliable as the sine method if proper steps are taken to correct for these influential factors. However, most observers are not likely to perform them in the field, especially if their correction would prove time-consuming. For the third tree in this study, the UAM shortleaf pine (Table 3), four of the six sine observations fell within 0.2 m of the maximum sine height estimate of 32.0 m, with the other two noticeably less (4.2 and 1.8 m, respectively). The tangent-based observations varied more, ranging from just under 31 m to almost 38 m, and mostly overestimated height. Two of the three points with the lowest tangent heights also produced the lowest sine heights, suggesting a subordinate crown position.

Cost Effectiveness

Because of the differences in how the technology is applied, it is hard to compare the sine and tangent methodologies under most field conditions. A clinometer ($100–200) and a logger's tape ($50) are the most affordable, and laser measuring devices can run from basic units ($200–400, without inclinometers and relatively low accuracy of ±1 m) to thousands of dollars. Devices capable of both fairly accurate distance and angle measurements can be purchased for $600 to 1,200. Although this amount may seem high, the greatest cost involved is the total time it takes to get an accurate measurement.

In most forested settings, the tangent method usually is quicker, if less accurate. The need for accurate height estimates must be balanced with the expenses of obtaining that accuracy. If the cost of having some degree of unpredictable errors in the height data does not exceed the expediency of using the tangent method, then few users are likely to switch to the slower but more accurate sine method. However, it should not be assumed that tangent height measurement errors are randomly distributed and unbiased, even over the course of a large inventory (i.e., as equally likely to overpredict and underpredict, with the total errors then "averaging" out) because technique biases and crown irregularities tend to overestimate height.

Conclusions

Height accuracy depends heavily on the assumptions violated when measuring a particular tree. Without the proper adjustments and identification of the true top of the tree, the similar triangle- or tangent-based methods will produce significant errors. The sine method, which directly measures the slope distance to a given part of the tree, automatically adjusts for tree lean, crown top offset, or sloping ground, assuming (as is the case with the tangent method) that all angles and distances are accurately measured.

With the sine method, having multiple height measurements for a single tree from different perspectives makes it easy to predict maximum height, because the highest value is the best estimate. However, there is no general rule that can be applied when using the tangent method. One thought has been to assume that estimation errors (both positive and negative) are equally distributed, and that an average of multiple readings would produce a relatively unbiased estimate of height. Table 3 shows that this is not necessarily so and that observer biases (e.g., most people would not measure toward or away from the lean of a tree) and the complexity of crown architecture tends to favor overestimates. Averaging may reduce the departure from actual height, but this is only true if no other systematic biases exist.

There are some challenges to using the sine method, not the least of which is that because it is not the default program of laser hypsometers, it often must be calculated manually. Some people will be reluctant to change how heights are measured, especially when they have many years of data using more conventional approaches. To them, the accuracy improvement would have to be dramatic for them to even consider the sine method. However, there are many circumstances where highly precise and accurate tree height measurements are required. For these, the sine method is superior to the tangent method. In the end, the user must balance their need for consistency and expedience with the desire for the best possible measure of tree height.

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